Risk index and uncertain portfolio Selection

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ABSTRACT

Portfolio selection is concerned with selecting an optimal portfolio that can strike a balance between maximizing the return and minimizing the risk among a large number of securities. Traditionally, security returns were regarded as random variables. However, the security market is complex and randomness is not the only type of uncertainty in reality. It is well known that the security returns are sensitive to various factors including economic, social, political and very importantly, people's psychological factors. It is found that future security returns, especially short term security returns are strongly affected by people's psychological expectation and are hard to be well reflected by the historical data. In this paper, we introduce a new type of variable to reflect the subjective imprecise estimation of the security returns. A risk index for uncertain portfolio selection is proposed and a new safe criterion for judging the portfolio investment is introduced. Based on the proposed risk index, a new risk index model is developed and its crisp form is given.

INTRODUCTION

Portfolio selection is concerned with selecting optimal combination of securities among a large number of candidate securities. Since Markowitz (Markowitz, 1952), quantitative analysis on portfolio selection has been a hot research topic. Traditionally, security returns used to be assumed to be random variables and a great deal of achievements have been made in portfolio theory based on this assumption, for example, recent works (Abdelaziz etc., 2007; Corazza etc., 2007; Huang, 2008b; Lin and Liu, 2008), etc. However, it is found that many security returns, especially short term security returns are hard to be well reflected by the historical data. The prediction of these returns relies heavily on the people's estimation. So the assumption of returns being random variables is questioned in this situation, and many scholars argued that we should find other ways to model the people's subjective imprecise estimation. With the introduction and development of fuzzy set theory, scholars have tried to employ fuzzy number and fuzzy set theory to manage portfolio since 1990s. For example, some researchers such as Watada (1997), Carlsson et al (2002), Lacagnina and Pecorella (2006) etc. employed possibility measure to study fuzzy portfolio selection problems, while Huang used credibility measure to develop a credibilistic VaR method (Huang, 2006}, mean-variance method (Huang, 2007) and mean-risk model method (Huang, 2008a). Qin et. al. (2009) proposed a fuzzy cross-entropy model, and Li et. al. (2010) developed a mean-variance-skewness model. A detailed survey about fuzzy portfolio selection based on credibility measure can be found in Huang (2009, 2010).

With the deeper research on portfolio selection, we found that paradoxes will appear if we use fuzzy variable to describe the subjective estimation of security returns. As we know, the estimation of a security return is calculated via the estimation of the security price. For example, if the estimation of a security price is regarded as a fuzzy number, then we have a membership function to characterize it. Suppose it is a triangular fuzzy variable $\xi = (1.5, 2.0, 2.5)$ dollars (see Fig. 1). Based on the membership function, it is known from possibility theory (or credibility theory) that *the price is exactly 2.0 dollars with belief degree 1 in possibility measure (or 0.5 in credibility measure)*. However, this conclusion is unacceptable because the belief degree of *exactly 2.0 dollars* is almost zero. In addition, the price being *exactly 2.0* and *not exactly 2.0* have the same belief degree in either possibility measure or credibility measure, which implies that the price being *exactly 2.0* and *not exactly 2.0* will occur equally likely. This conclusion is quite astonishing and hard to accept. Recently, Liu (2007) proposed an uncertain measure and developed an uncertainty theory which can be used to handle subjective imprecise quantity. Much research work has been done on the development of uncertainty theory and related theoretical work. For example, Liu discussed uncertain calculus (Liu, 2009a) and uncertain programming(Liu, 2009b). Gao (2009) discussed some properties of continuous uncertain measure. You (2009) proved some convergence theorems of uncertain sequences. Li and Liu (2009) discussed uncertain logic. Liu (2008) defined uncertain process, and Chen and Liu (2010) proved the existence and uniqueness theorem for uncertain differential equations, etc. When we use uncertainty theory to model subjective estimation of security returns, the above mentioned paradoxes will disappear immediately. In this paper, we will use uncertain variable to describe the experts' estimation of security returns and use uncertain measure to reflect the belief degree of an uncertain event. Furthermore, we will define a risk index and propose a risk index model for portfolio selection problem with uncertain returns.

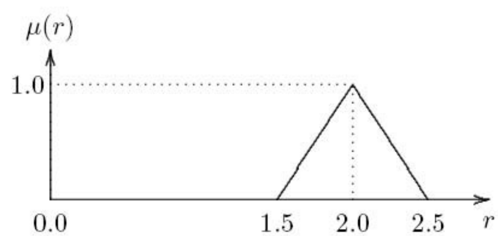

Figure 1: Membership function of a security price $\xi = (1.5, 2.0, 2.5)$

The rest of the paper is organized as follows. For better understanding of the paper, some necessary knowledge about uncertain variable will be introduced in Section 2. Then risk index will be proposed in Section 3 and a risk index model will be developed in Section 4. After that, the crisp form of the model will be presented in Sections 5. Finally, in Section 6, some conclusion remarks will be given.

NECESSARY KNOWLEDGE ABOUT UNCERTAIN VARIABLE

To better describe the subjective imprecise quantity, Liu (2007) in 2007 proposed an uncertain measure and further developed an uncertainty theory which is an axiomatic system of normality, monotonicity, self-duality, countable subadditivity and product measure.

Definition 1 (Liu, 2007) Let Γ be a nonempty set, and L a σ-algebra over Γ. Each element $\Lambda \in L$ is called an event. A set function $M\{\Lambda\}$ is called an uncertain measure if it satisfies the following four axioms:

(i) (Normality) $M\{\Gamma\}=1$.

(ii) (Monotonicity) $M\{\Lambda_1\} \le M\{\Lambda_2\}$ whenever $\Lambda_1 \subseteq \Lambda_2$.

(iii) (Self-Duality) $M\{\Lambda\}+M\{\Lambda^c\}=1$.

(iv) (Countable Subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have $M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}$. The triplet (Γ, L, M) is called an uncertainty space.

In order to define product uncertain measure, Liu (2009) proposed the fifth axiom as follows:

(v) (Product Measure) Let (Γ_k, L_k, M_k) be uncertainty spaces for $k=1,2,\cdots,n.$ The product uncertain measure is $M = M_1 \wedge M_2 \wedge \cdots \wedge M_n$.

Definition 2 (Liu, 2007) An uncertain variable is a measurable function ξ from an uncertainty space (Γ, L, M) to the set of real numbers, i.e., for any Borel set of B of real numbers, the set $\{\xi \in B\}=\{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event.

In application, a random variable is usually characterized by a probability density function or probability distribution function. Similarly, an uncertain variable can be characterized by an uncertainty distribution function.

Definition 3 (Liu, 2007) The uncertainty distribution $\Phi: R \to [0,1]$ of an uncertain variable ξ is defined by $\Phi(t)=M\{\xi \le t\}$.

For example, by a normal uncertain variable, we mean the variable that has the following normal uncertainty distribution

$$\Phi(t)=\left(1+\exp\left(\frac{\pi(\mu-t)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad t \in R$$

where μ and σ are real numbers and $\sigma>0.$ For convenience, it is denoted in the paper by $\xi \sim N(\mu,\sigma).$ It has been proven (Liu, 2010) that if $\xi_i \sim N(\mu_i,\sigma_i), i=1,2,\cdots,n,$ are normal uncertain variables, then $\eta=\sum_{i=1}^{n}\lambda_i\xi_i$ is also a normal uncertain variable

$\eta \sim N(\sum_{i=1}^{n}\lambda_i\mu_i, \sum_{i=1}^{n}\lambda_i\sigma_i)$ for $\lambda_i>0, i=1,2,\cdots,n.$

We call an uncertain variable the linear uncertain variable if it has the following linear uncertainty distribution

$$\Phi(t)=\begin{cases} 0, & \text{if } t<a \\ t-a/b-a, & \text{if } a \le t \le b \\ 1, & \text{if } t>b. \end{cases}$$

For convenience, it is denoted in the paper by $\xi \sim L(a,b)$ where $a<b$. It has been proven (Liu, 2010) that if $\xi_i \sim L(a_i,b_i)$ are linear uncertain variables, then $\eta \sim L(\sum_{i=1}^{n}\lambda_i a_i, \sum_{i=1}^{n}\lambda_i b_i)$ is also a linear uncertain variable $\eta \sim L(\sum_{i=1}^{n}\lambda_i a_i, \sum_{i=1}^{n}\lambda_i b_i)$ for $\lambda_i > 0, i=1,2,\cdots,n.$

When the uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ are represented by uncertainty distributions, the operational law is given by Liu (2010) as follows:

Theorem 1 (Liu, 2010) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. Let $f(t_1, t_2, \cdots, t_n)$ be strictly increasing with respect to $t_1, t_2, \cdots, t_n$. Then $\xi = f(t_1, t_2, \cdots, t_n)$ is an uncertain variable with uncertainty distribution

$$\Psi(t) = \sup_{f(t_1,t_2,\cdots,t_n)=t} (\min_{1\le i\le n} \Phi_i(t_i)), \quad t \in R \qquad (1)$$

whose inverse function is

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)), \quad 0<\alpha<1 \qquad (2)$$

if $\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)$ are unique for each $\alpha \in (0,1)$.

To tell the size of an uncertain variable, Liu defined the expected value of uncertain variables.

Definition 4 (Liu, 2007) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{\infty} M\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} M\{\xi \le r\}\mathrm{d}r \qquad (3)$$

provided that at least one of the two integrals is finite.

RISK INDEX

In portfolio selection, how to define risk is one of the most important topics. The earliest and the most popular risk definition is variance. It was given by Markowitz (1952) in 1952. He proposed that the expected value of a portfolio return could be regarded as the representative of the investment return and variance the risk of the investment. The idea is that the greater deviation from the expected value, the less likely the investors can obtain the expected return, and thus the riskier the portfolio. Therefore, for conservative investors, when making investment, they should first require that the portfolio be safe enough, i.e., the variance value of the portfolio be less than or equal to a predetermined tolerable variance level and then to select among the safe portfolios the one with maximum expected return. In mathematical way, the mean-variance model is expressed as follows:

$$\begin{cases} E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\ s.t. \\ \quad V[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \le c \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, i = 1,2,\cdots,n. \end{cases} \qquad (1)$$

where E denotes the expected value operator, V the variance operator, ξ the investment proportion in the i-th security, ξ_i the random returns for the i-th securities, $i=1,2,\cdots,n$,

Table 1: Random returns of portfolios A and B

Portfolio	Random Return Interval	Variance
A	[0,1]	1/12
B	[100,101]	1/12

respectively, and c the predetermined maximum risk level (i.e., the predetermined maximum tolerable variance value).

Though variance is a popular risk measure, it is not so convenient to use for investors. It is seen from the model (4) that before knowing the expected return of the portfolio, the investors need to give a risk level, i.e., the maximum tolerable variance level. However, it is difficult to judge if a variance level is risky or not when the expected value of the portfolio is unknown. For example, suppose we have two portfolios A and B. The random return of portfolio A is uniformly distributed on [0,1] and the random return of B on [100,101] (see Table1). It is easy to see that the variance values of portfolios A and B are same because the return of Portfolio B is just a movement of Portfolio A from lower bound 0 to 100 and upper bound 1 to 101. However, Portfolio B will quite likely be regarded as safe but Portfolio A be regarded risky. To solve the problem, utility function is proposed which assigns real numbers to the expected and variance values respectively in a way that captures the investors' preferences over tradeoff of investment return and risk. However, determination of the utility function remains a difficult task. This is also true in uncertain portfolio selection. These difficulties motivate the author to propose an easier-to-use risk measure. As we know, people can choose to invest their money in risk free asset and gain risk free interest rate with certainty. Therefore, any returns below the risk free interest rate will be regarded as losses. To obtain an average level of the portfolio return below the risk free interest rate, we define a risk index as follows:

Definition 5 Let ξ denote an uncertain return rate of a security, and r_f the risk-free interest rate. Then the risk index of the portfolio is defined by

$$RI(\xi) = E[(r_f - \xi)^+],$$

where

$$(r_f - \xi)^+ = \begin{cases} r_f - \xi, & \text{if } r_f - \xi \geq 0 \\ 0, & \text{otherwise.} \end{cases} \qquad (5)$$

Let ξ be an uncertain security return with continuous uncertainty distribution Φ. Then the risk index of the security can be expressed as follows:

$$\begin{aligned} RI(\xi) &= E[(r_f - \xi)^+] \\ &= \int_0^{\infty} M\{r_f - \xi \geq r\}\mathrm{dr} \\ &= \int_0^{\infty} M\{\xi \leq r_f - r\}\mathrm{dr} \\ &= \int_0^{r_f} M\{\xi \leq t\}\mathrm{dt} \\ &= \int_0^{r_f} \Phi(t)\mathrm{dt}. \qquad (6) \end{aligned}$$

Example 1 Suppose that the security return is a linear uncertain variable $\xi \sim L(a,b)$ where $a < r_f$. Then the risk index of the security is as follows:

$$RI(\xi)=\int_a^{r_f}(r-a)/(b-a)\mathrm{d}r=(r_f-a)^2/2(b-a). \qquad (7)$$

Theorem 2 Let ξ be an uncertain security return with continuous uncertainty distribution Φ whose inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha\in(0,1)$. Then the risk index of the security can be expressed as follows:

$$RI(\xi)=\int_0^{\beta}(r_f-\Phi^{-1}(\alpha))\mathrm{d}\alpha, \qquad (8)$$

where β is defined by $\Phi^{-1}(\beta)=r_f$.

Proof: It follows directly from the equation (6). See Fig.2.

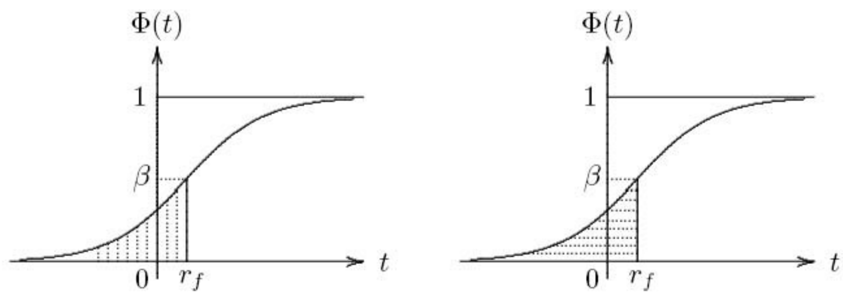

Figure 2: Risk Index via integral

Example 2 Suppose that the security return is a normal uncertain variable $\xi\sim N(\mu,\sigma)$. Then the risk index of the security is as follows:

$$RI(\xi)=\int_0^{\beta}\left(r_f-\mu-\frac{\sqrt{3}\sigma}{\pi}\ln\frac{\alpha}{1-\alpha}\right)\mathrm{d}\alpha=\beta(r_f-\mu)-\frac{\sqrt{3}\sigma}{\pi}[\beta\ln\beta+(1-\beta)\ln(1-\beta)] \qquad (9)$$

where $\beta=\exp\left(\frac{\pi(r_f-\mu)}{\sqrt{3}\sigma}/(1+\exp(\frac{\pi(r_f-\mu)}{\sqrt{3}\sigma})\right)$.

Since the risk free interest rate r_f is known before making the decision, it is much easier for the investors to tell how much level below r_f they can tolerate. Thus, given *c* the value that the investors can tolerate below the risk free interest rate, a portfolio is regarded to be safe if

$$RI(\xi)\le c. \qquad (10)$$

RISK INDEX MODEL

When making investment, the investors will usually require that the portfolio be safe enough and then pursue the maximum return. Since the optimal investment return may not be obtained in some adverse situation, it is natural that people would accept the inability to reach the objective to some extent. However, at a given confidence level which is considered as the safety margin, the objective must be achieved. Let x_i denote the investment proportions in securities *i*, ξ_i the uncertain returns for the *i*-th securities, $i=1,2,\cdots,n$, respectively, γ the predetermined confidence level the investor accepts, and *c* the investor's tolerable value below the risk free interest rate. To the idea can be expressed mathematically as follows:

$$
\begin{cases}
\max \bar{f} \\
s.t. \\
\quad M\{x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \geq \bar{f}\} \geq \gamma \\
\quad RI(x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) \leq c \\
\quad x_1 + x_2 + \cdots + x_n = 1 \\
\quad x_i \geq 0, i = 1,2,\cdots,n
\end{cases} \tag{11}
$$

where RI is the risk index of the portfolio defined as

$$RI(x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) = E[(r_f - (x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n))^+]$$

and $\bar{f}$ is the γ-return defined as

$\max\{\bar{f} \mid M\{x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \geq \bar{f}\} \geq \gamma\}$ which means the maximal investment return the investor can obtain at confidence level γ. It is clear that portfolios whose risk index is not greater than the preset level are safe portfolios, and among them the portfolio with the maximum $\bar{f}$ is the optimal portfolio the investor should select.

Crisp Form

When the uncertainty distributions of the security returns are known, the model (11) can be transformed into the following crisp form.

Theorem 3 Let Φ_i denote the continuous uncertainty distribution of the i-th uncertain security return rate ξ_i whose inverse function $\Phi_i^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1), i = 1,2,\cdots,n$, respectively. Then the risk index model (11) can be transformed into the following form:

$$
\begin{cases}
\max x_1\Phi_1^{-1}(1-\gamma) + x_2\Phi_2^{-1}(1-\gamma) + \cdots + x_n\Phi_n^{-1}(1-\gamma) \\
s.t. \\
\quad \beta r_f - \int_0^\beta (x_1\Phi_1^{-1}(\alpha) + x_2\Phi_2^{-1}(\alpha) + \cdots + x_n\Phi_n^{-1}(\alpha))\mathrm{d}\alpha \leq c \\
\quad x_1\Phi_1^{-1}(\beta) + x_2\Phi_2^{-1}(\beta) + \cdots + x_n\Phi_n^{-1}(\beta) = r_f \\
\quad x_1 + x_2 + \cdots + x_n = 1 \\
\quad x_i \geq 0, i = 1,2,\cdots,n.
\end{cases} \tag{12}
$$

Proof: Let Ψ denote the uncertainty distribution of the portfolio $\sum_{i=1}^{n} x_i\xi_i$. Since Φ_i are continuous uncertainty distribution functions, it follows from the self-duality property of the uncertainty measure and the definition of uncertainty distribution that $\bar{f} = \Psi^{-1}(1-\gamma)$. It follows from Theorem 1 that

$\Psi^{-1}(1-\gamma) = x_1\Phi_1^{-1}(1-\gamma) + x_2\Phi_2^{-1}(1-\gamma) + \cdots + x_n\Phi_n^{-1}(1-\gamma)$.

It is known from Theorem 2 that

$RI(x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) = \int_0^\beta \Psi^{-1}(\alpha)\mathrm{d}\alpha$, where β is determined by $\Psi^{-1}(\beta) = r_f$. Then we can get Theorem 3 directly from Theorem 1. Thus the theorem is proven.

Example 3. Suppose the return rates of the *i*-th securities are all normal uncertain variables $\xi_i \sim N(\mu_i, \sigma_i), i = 1, 2, \cdots, n$, respectively. According to Theorem 3, the risk index model can be transformed into the following form:

$$\begin{cases} \max \sum_{i=1}^{n} \left(x_i \mu_i - \frac{\sqrt{3} x_i \sigma_i}{\pi} \ln \frac{\gamma}{1-\gamma} \right) \\ s.t. \\ \quad \beta(r_f - \sum_{i=1}^{n} x_i \mu_i) - \frac{\sqrt{3}}{\pi} \sum_{i=1}^{n} x_i \sigma_i [\beta \ln \beta + (1-\beta) \ln(1-\beta)] \le c \\ \quad \beta = \exp\left(\pi (r_f - \sum_{i=1}^{n} x_i \mu_i) / \sqrt{3} \sum_{i=1}^{n} x_i \sigma_i \right) \Big/ \left(1 + \exp\left(\pi (r_f - \sum_{i=1}^{n} x_i \mu_i) / \sqrt{3} \sum_{i=1}^{n} x_i \sigma_i \right) \right) \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, i = 1, 2, \cdots, n. \end{cases} \tag{13}$$

Example 4. Suppose the return rates of the *i*-th securities are all linear uncertain variables $\xi_i \sim L(a_i, b_i), i = 1, 2, \cdots, n,$ respectively. According to Theorem 3, the risk index model can be transformed into the following form:

$$\begin{cases} \max \sum_{i=1}^{n} \left(\gamma a_i x_i + (1-\gamma) b_i x_i \right) \\ s.t. \\ \quad (r_f - \sum_{i=1}^{n} a_i x_i)^2 / 2(\sum_{i=1}^{n} b_i x_i - \sum_{i=1}^{n} a_i x_i) \le c \\ \quad x_1 + x_2 + \cdots + x_n = 1 \\ \quad x_i \ge 0, i = 1, 2, \cdots, n. \end{cases} \tag{14}$$

CONCLUSIONS

This paper has discussed the portfolio selection problem when security returns are given by experts' estimation rather than historical data. The paper regards the security returns as uncertain variable. After discussing the limitation of variance as a risk measure in application, the paper has introduced a risk index as an alternative risk measurement which is easier to use than variance. Based on the new risk measure, a risk index model has been developed. In addition, the crisp form of the model has also been provided.

ACKNOWLEDGEMENT

This work was supported by National Natural Science Foundation of China Grant No.70871011, Program for New Century Excellent Talents in University, and the Fundamental Research Funds for the Central Universities.

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