A Graph-Theoretic Approach to Improved Curriculum Structure and Assessment Placement

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ABSTRACT

Assurance of student learning is an important issue in modern education. Accrediting bodies and government entities alike have begun requiring proof that students actually learn the material taught in the classroom. To comply with this mandate, education programs are actively engaged in incorporating assessment procedures into their curricula. Unfortunately, there are no generally agreed upon methods on how to do this. This paper addresses the issue and answers several key questions concerning where to locate topic coverage, assessment data collection, and corrective action within the curriculum. The approach described and developed in this paper cast the curriculum problem into an abstract graph representation. Graph-theoretic metrics are calculated and visualization software is utilized to create a picture that helps answer the research questions. The result is a systematic, easy to understand approach that produces defensible output which should greatly aid faculty and administrators tasked with creating an assessment scheme.

INTRODUCTION

Assurance of student learning is now required by many accrediting bodies as a key component of initial and reaccreditation requests (AACSB, 2010; SACS, 2010; MSC, 2009; HLC, 2009). The United States government has also embraced the practice as evidenced by the final version of the Spellings Report (Spellings, 2006). This report was commissioned by the U.S. Department of Education as a roadmap for the future of higher education in the United States. In this report, the Commission recommended the following:

> *To meet the challenges of the 21st century, higher education must change from a system primarily based on reputation to one based on performance. We urge the creation of a robust culture of accountability and transparency throughout higher education*
> (Spellings 2006, p. 21).

This aspiration is shared by state governments and school districts across the nation (No Child Left Behind, 2009; Jones, 2007). Clearly, the mood of the tax-paying population and the representatives they elect is one of educational accountability and outcome-based results.

In an attempt to comply with this mandate, colleges and universities are rushing to incorporate direct program assessment processes into their curricula. For some institutions this is a fairly minor task because outcome-based performance measures and assessment were already a part of the academic culture. For others, the required inclusion of assessment is a massive undertaking of program redesign, process creation, data collection, and corrective analysis. In both cases, decisions must be made involving curriculum redesign and assessment structure. For example, institutions must decide what goals and objectives are appropriate for programs, what topics should be covered to achieve objectives, where the topics should be taught, where objective assessment should be performed, what is the threshold of a "successful" assessment, etc. These questions must be answered and the solutions implemented quickly if the institution is to gain or maintain its accreditation. Unfortunately, program redesign and haste are incompatible concepts even under ideal conditions. Given the current state-of-the-art of program redesign and the paucity of research applicable to assessment inclusion, conditions are far from ideal for most institutions.

The purpose of this research is to address this problem by answering some of the key questions involved in incorporating formal assessment into an existing curriculum. Specifically, this paper will deal with the following questions.

- Where in the curriculum should introductory topic coverage be placed?
- Where should reinforcement of assessment topics take place?
- Where should primary and secondary objective assessment be located?
- Where should changes identified by assessment feedback be implemented?

These are fundamental structural questions that must be answered by the faculty and administrators tasked with assessment inclusion. Currently, there are no generally agreed upon methods to deal with these structural concerns. Without a systematic, repeatable methodology to deal with these issues, the resulting assessment scheme can be arbitrary, inconsistent, and ineffective. Given the public interest in educational accountability and outcome-based learning, this is not good enough.

The approach developed and described by this paper will use graph-theoretic metrics and visualization to analyze an existing curriculum structure. The graph visualizations will show the courses, pre-requisite chains, program memberships, and course types of the curriculum. From these networks, basic graph theory metrics will be calculated and used to indicate the appropriate location for assessment components and modifications within the curriculum. While the locations determined are not proven to be optimal within the curriculum, they are based on a theoretical background that is systematic, defensible, and repeatable. These characteristics make it an excellent choice to establish the initial structural aspects of a new assessment scheme or to fine tune an existing one.

GRAPH-THEORETIC BACKGROUND

Graph theory is a branch of mathematics that studies the connections, structure, and properties of graphs. A *graph* is merely an abstract construct consisting of nodes (i.e., *vertices*) and connecting lines (i.e., *edges*). Each edge connects exactly two vertices. The graph can be *undirected* or *directed*. An undirected graph is one where there is no particular ordering to the connected vertices; they are simply connected by an edge. A directed graph is one where the edges imply a specific ordering, or direction, to the edge connection. The implication of this is that one vertex of the pair occurs first and the other second. A *weighted* graph is one where a value is associated with the edges and/or vertices. Within the graph, the number of vertices is called its *order* and any graph with an order of 0 or 1 is generally denoted as *trivial*. The number of edges directly connecting to a vertex is called its *degree* and a vertex with a degree of 0 is said to be *isolated*. Finally, a vertex is *incident* to an edge if that edge connects it to another vertex; whereas, two vertices are *adjacent* if they both have the same incident edge in common (Newman, 2007; Diestel, 2000).

The historical origins of graph theory lie in a paper published in 1736 by Leonhard Euler concerning an algorithm to cross all seven bridges in the city of Königsberg in Prussia (Biggs, Lloyd, & Wilson, 1998, chpt. 1). The constraint on this problem was that all seven bridges be crossed exactly one time. Euler's abstract representation of the problem as vertex pairs connected by edges and his algorithmic solution laid the foundations for both graph theory and topology. Since his seminal work, the use of graph theory has expanded to many diverse fields including anthropology, architecture, biology, chemistry, physics, circuit design, disease transmission prediction, and social network analysis (Buckley & Harary, 1990).

Basic Graph Metrics

Graph theory is vast field of study that includes a rich collection of research and applications. Not all of graph theory is applicable to the current research project; consequently, this paper will only consider the relatively small subset related to structural centrality, clustering density, and degree. These concepts have been used for a number of important analysis purposes; the most notable recent application being the analysis of social networks (Smith, Hansen, & Gleave, 2009; Shamma, Kennedy, & Churchill, 2009; Borgatti,2005a; Brandes, 2001). These metrics are particularly applicable to the curriculum design problem because they capture the key structural elements needed to determine placement of topic introduction, reinforcement, and assessment. The remainder of this section will introduce the basic metrics used in this research. Formal mathematical details are omitted because they would unnecessarily complicate the presentation of the material.

Measures of Degree

The *degree* of a vertex is simply a count of the number of edges incident to that vertex. In other words, the degree of a vertex is the number of edges connected to it. From an analysis standpoint, vertices with a high degree are an indication of key points of exposure for whatever is flowing through the network and the opportunity to influence or augment this flow (Borgatti, 2005b). It is also considered an important measure of the influence or importance of a vector in the structure (Newman, 2007). There is only one measure of degree in an undirected graph. If the edges in the graph are directed, there are two measures of degree: *in-degree* and *out-degree*. As the names imply, in-degree is a count of the number of edges coming into a vertex whereas out-degree is the number of edges going out of a vertex. Analysis of the in-degree measure indicates the destination of the flow while out-degree analysis implies the origin. Figure 1 shows an undirected graph with 10 vertices and 17 edges (Hansen, Shneiderman, & Smith, 2009). The degree of vertex E is 3 and the degree of vertex D is 6.

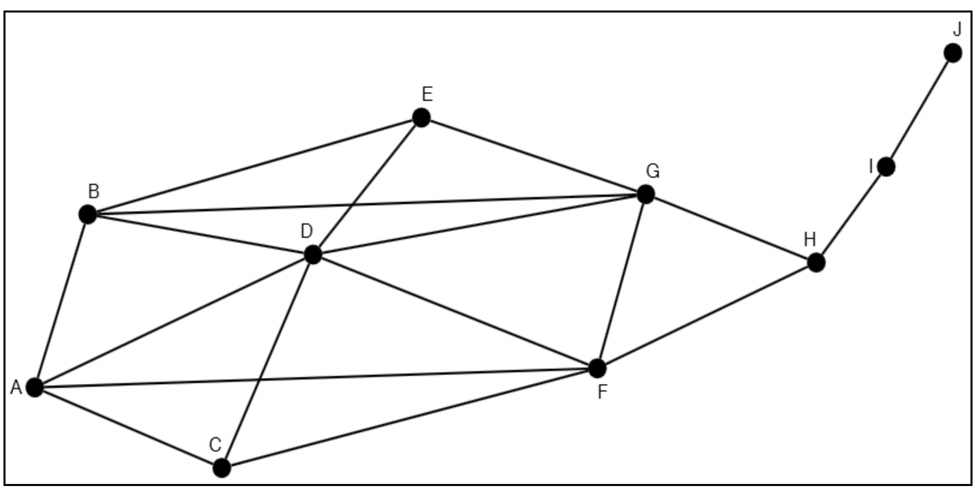

Figure 1: Undirected graph

Measures of Centrality

The notion of graph centrality is very important to the study of graph structure because it implies locations of power, prestige, prominence, and importance (Borgatti, 1995; Freeman, 1979). This is particularly true in organizational graph structures where some sort of flow is implied (Borgatti, 2005a).

The key question considered by centrality measures is, "What is the most important vertex in the graph?" The answer to this question depends upon how one defines "important" (Newmann, 2007). For the purposes of this research, two fundamental measures of centrality are utilized: betweenness centrality and eigenvector centrality. Both measures provide a slightly different interpretation of what constitutes an "important" vector; consequently, each can be used to deal with a slightly different structural concern.

Betweenness centrality is useful in finding vertices that serve as a bridge from one part of a graph to another. Consequently, betweenness is a rudimentary measure of the control that a specific vertex exerts over the flow throughout the full graph (Newman, 2007). For example, vector H in Figure 1 is the connection between the main part of the graph and vectors I and J. Were vector H to be removed, I and J would be cut off from the rest of the graph. This makes H "important" because it ensures that no nodes are isolated. This is not true for vertex G even though G has a much higher degree. When using betweenness centrality as an analysis measure, it indicates a potential gate keeping or controlling vector.

Eigenvector centrality can best be thought of as an indirect measure of centrality. A key difference between this measure and the less sophisticated centrality metrics is that eigenvector centrality recognizes that some connections are more important (or influential) than others (Newman, 2007; Bonaciche, 1972). Consequently, it considers not only the degree of a vector but also the degree of all the vectors that connect to it (Hansen, Shneiderman, & Smith, 2009; Alexander, 1963). The implication is that proximity adjacent to a highly connected vector is an important characteristic to note due to the influence it potentially affords the vector (Borgatti, 1995). Figure 1 illustrates this as follows: vectors H and E have identical degree values; however, vector E is connected to vector D which has the highest degree in the graph. Because of this, E is more likely to be influential and able to affect change within the graph than H. As would be expected, E has the higher eigenvector value.

Clustering Measures

The *clustering coefficient* is useful in determining how tightly adjacent vectors cluster together. In this case, the word "tight" does not imply spatial distance, but rather the density of existing edge connections as compared to the total possible connections. The value of the coefficient ranges from 0 to 1 with 1 being the value of a *complete graph* (Watts & Strogatz, 1998). A complete graph is one where every vertex is connected to every other vertex by an edge (Diestel, 2000). This measure is useful because it helps identify key vertices with enhanced importance or influence within a densely connected group. These vertices would be useful in efficiently implementing change into the cluster. Vertices with a high clustering coefficient also identify the key points of a *clique*—a subgraph of densely connected vertices (Melnikov, Sarvanov, Tyshkevich, Yemelichev, & Zverovich, 1998). To illustrate this metric, vectors B, D, and G in Figure 1 are all connected to one another by the maximum number of edges; which is 3. They form a complete graph and hence would have a high clustering coefficient of 3/3 or 1. On the other hand, vectors F, G, and I are not densely connected (there is 1 connection out of a possible 3); hence, the clustering coefficient is 1/3. This coefficient denotes a lack of dense clustering which, in turn, implies the absence of a central point of influence.

GRAPH THEORY APPLIED TO CURRICULUM DESIGN

The curriculum of an educational program can be represented as a graph. This type of abstraction from real-world problem to graph construct is very common and has been applied to numerous problems including: package delivery, disease propagation, e-mail transmission, money exchange, and communication in social networks (Borgatti, 2005a). For the curriculum context, courses become vectors and the edges correspond to prerequisite requirements. Vectors that are not part of a prerequisite chain would be isolated. Since a prerequisite relationship has a distinct ordering, the graph would need to be a *directed* graph. Finally, assuming that the course is passed, standard curriculum schemes do not require that a course be taken multiple times to satisfy graduation requirements. This means that only one edge would be incident to any two vectors and no self-loops would be present. Thus, the resulting curriculum graph would be classified as *simple* and *acyclic*. From these definitions, it follows that a typical academic curriculum can be represented as a *simple acyclic directed graph*. A representation of this type of graph is shown in Figure 2.

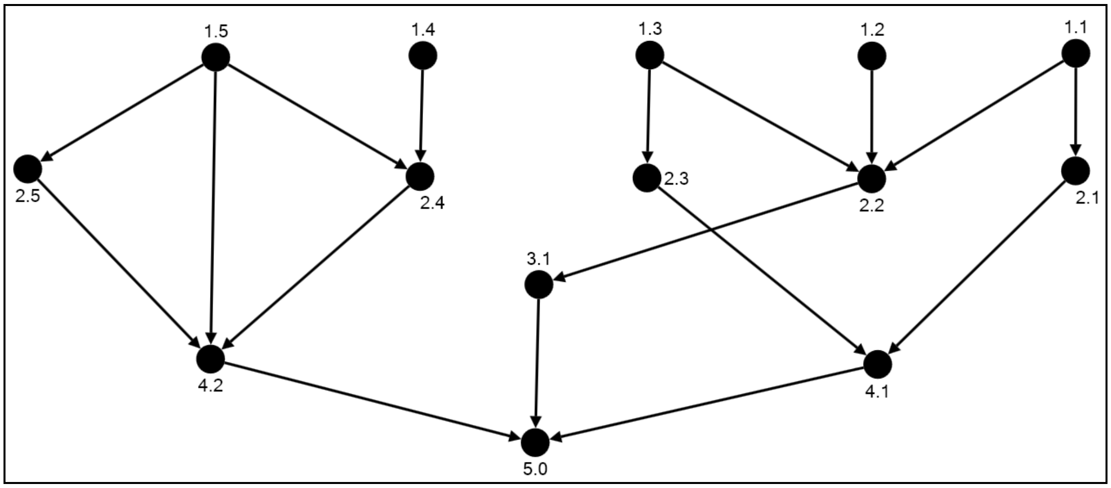

Figure 2: Simple curriculum graph

In Figure 2, the course vertices are represented by numbered nodes. The numbering scheme was selected because it implies the progression from the freshman-level courses to senior-level courses. The directed edges between the vertices represent the prerequisite requirements for courses. Thus, the course represented by vertex 2.2 has prerequisites of 1.1, 1.2, and 1.3. From a graph metric standpoint, vector 2.2 has an in-degree of 3 and an out degree of 1.

The graph in Figure 2 can be converted into a mathematical format that allows for calculation and manipulation. Numerous software packages have been developed specifically to handle these calculations and provide a visual representation of the resulting graph. For the purposes of this research, the NodeXL add-in for Microsoft Excel provided by the CodePlex Open Source Community is utilized. NodeXL was selected because it is a freely available add-in that uses the familiar interface and functions built into Microsoft Excel. In addition, the add-in automatically performs the calculations needed for basic graph analysis and has a visualization component that allows the user to display the resulting graph as a picture. Components of this picture (e.g., the size of the vertices, the color of the edges, edge patterns) can be modified automatically by the software to indicate the values of common graph metric calculations. This is a powerful analytic capability that allows the user to visually emphasize the key structural elements and important connections in a curriculum based upon the graph-theoretic metrics of its graph. The remainder of this section demonstrates this capability on the simple curriculum in Figure 2 for the basic graph metrics described earlier.

Out-Degree

The out-degree of a vertex is a count of the number of directed incident edges that leave the vertex. The measure indicates the origin of flow within the directed graph and highlights points where that flow can be augmented or influenced (Borgatti, 2005b). When applied to a curriculum context, a vertex with a high out-degree indicates a course that is a prime location to expose students to introductory material or framework concepts. It also identifies a good course to perform baseline assessment to determine a student's initial knowledge of assessed topics. Figure 3 shows the curriculum modified to visually represent the out-degree metric of each vertex.

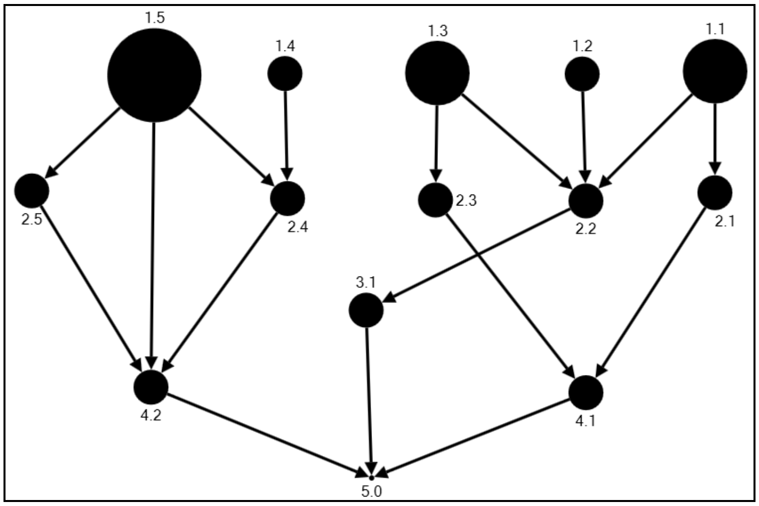

Figure 3: Out-degree of the simple curriculum

In Figure 3, the NodeXL add-in was used to calculate the out-degree metric for all nodes in the graph. These values were then used to automatically scale the size of the vertices so that nodes with a low out-degree are shown very small and nodes with a high out-degree are shown very large. For example, vertex 5.0 has an out-degree of 0; consequently, it is the smallest node in the graph. Vertex 1.5 has the largest out-degree of any vertex in the curriculum (i.e., 3) and is shown as the largest vertex on the graph. The other nodes vary in size depending upon the metric value. Should the difference in vector size not be as visually dramatic as shown in Figure 3, the user can always look at the actual numeric values calculated by NodeXL for comparison.

Based upon the out-degree values, the interpretation of Figure 3 indicates that courses 1.1, 1.3, and 1.5 are the best places to introduce topics and perform baseline assessment. On the other hand, course 5.0 would be the worst place. More complicated (and realistic) curriculum graphs would likely also show courses midway through the curriculum where it would be appropriate to introduce framework concepts that are built upon by later coursework.

In-Degree

The in-degree of a vertex is a count of the number of directed incident edges that come into the vertex. It is an indication of the destination of the network flow and is a key point of exposure for the end result of that flow. Figure 4 shows the curriculum recalculated to emphasize the in-degree metric for each vertex.

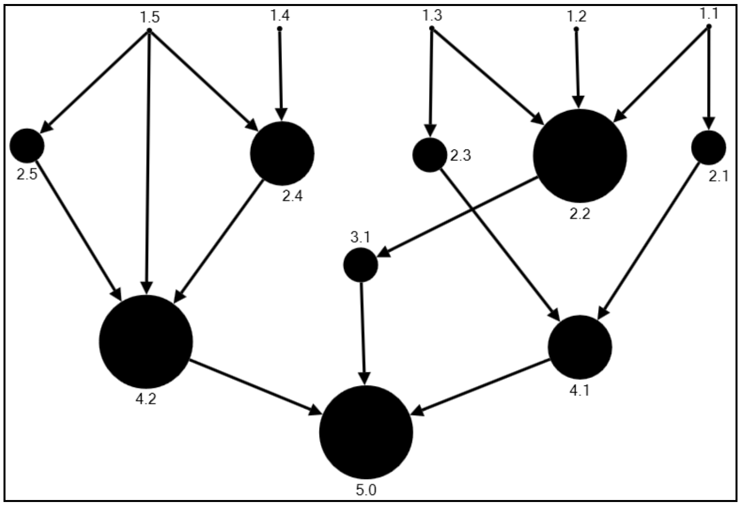

Figure 4: In-degree of the simple curriculum

In the curriculum context, the in-degree metric indicates courses where students should be engaged in higher-level learning activities; for example, analysis, synthesis, or evaluation tasks as defined by Bloom's taxonomy (Bloom, 1956). It also locates courses appropriate for final assessment of the program or a track within the program.

From Figure 4, it appears that courses 2.2, 4.2, and 5.0 are good prospects for assessment of student learning—courses 2.2 and 4.2 for intermediate topic assessment, and 5.0 for full program assessment. While course 5.0 is an obvious choice, 4.2, and 2.2 are less obvious and are the first indication that the method described by this research can provide an analytical basis for more intelligent and effective assessment placement than would be done otherwise.

Betweenness Centrality

The betweenness centrality measure is useful in finding vectors that form a bridge from one part of a graph to another. It indicates a crude measure of the control that a vector has over the flow through a connected graph (Newman, 2007). When used in a curriculum context, the course with the largest betweenness centrality value indicates a key link between program tracks or course clusters within the program. Figure 5 shows this for the simple curriculum.

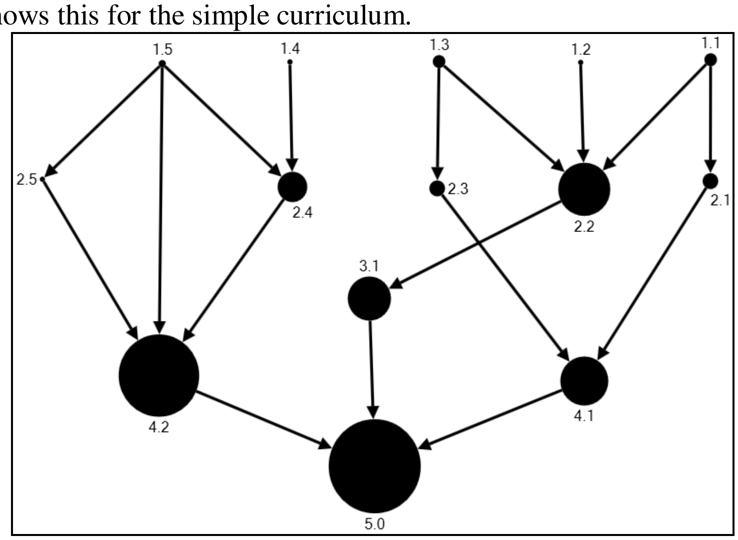

Figure 5: Betweenness centrality of the simple curriculum

Courses 4.2 and 5.0 have the largest values for betweenness centrality within the curriculum. The implication of this is that these courses bridge the gap between the structural partitions within the program. In the simple curriculum, courses 1.4, 1.5, 2.4, 2,5, and 4.2 on the left side of the graph form a relatively tightly connected course cluster. Within this cluster, 4.2 is the key course that links the group to the remainder of the program. Because of its structural position, it is an excellent candidate for assessment of the topics covered in the cluster. The same can be said for course 5.0 in relation to the full program because it links the left course cluster with the cluster on the right of the graph. Course 4.2 could also serve as a good location to reinforce assessment topics prior to the capstone assessment performed in course 5.0.

It is important to note at this time that the graph metrics calculated on a curriculum structure should not be considered in isolation. A quick comparison of Figures 4 and 5 reveals that courses 4.2 and 5.0 have been singled out by both the in-degree metric and the betweenness centrality measure as good candidates for assessment. This type of metric reinforcement is a major strength of the graph-theoretic approach described by this paper. When combined with the actual context of the academic program (i.e., knowledge about the actual courses and the material they cover), the approach gives the end-user a powerful tool for systematic curriculum design and assessment placement.

Eigenvector Centrality

The eigenvector centrality measure identifies those vectors that are adjacent to other vectors with high degree values. Consequently, it locates vectors in the structural position to yield influence disproportionate to the number of direct connections present (Borgatti, 1995). This is valuable information when applied to the curriculum context because it identifies courses that can act as reinforcement points prior to assessment. If the course is located early in the curriculum, it can also serve as a point to introduce topics and perform baseline assessment. Figure 6 shows the curriculum graph after modification with eigenvector centrality values.

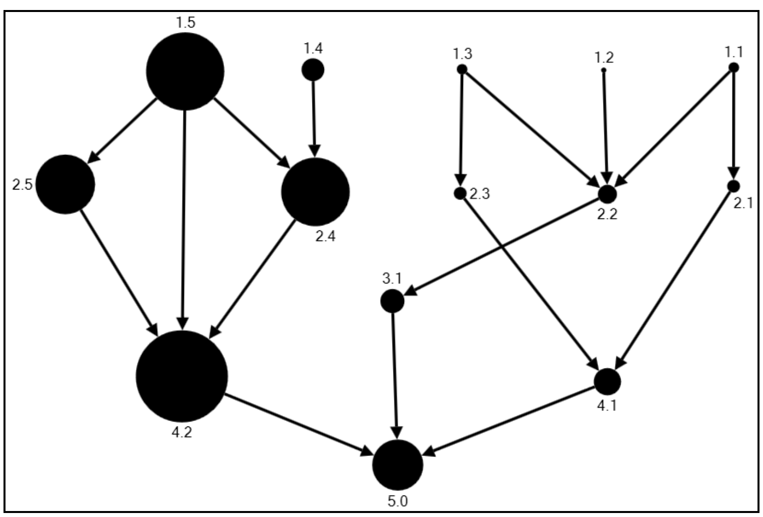

Figure 6: Eigenvector centrality of the simple curriculum

Figure 6 shows that courses 1.5 and 4.2 have the largest eigenvector values. The first implication of this is that 4.2 should be designed to reinforce topics that will be assessed in the 5.0 capstone course. This is logical given its proximity directly before 5.0 and after the other courses in its local cluster. The second implication is that the course labeled 1.5 should introduce topics that will be assessed later and should also be the site for baseline topic assessments to determine initial student performance levels. This conclusion is reached by considering both the eigenvector value and the out-degree of course 1.5. Taken together, the structural implications of the metric values suggest that this course holds a very strategic position in the overall curriculum assessment scheme.

Clustering Coefficient

The clustering coefficient is useful in determining the primary vector of influence in a densely connected cluster of vertices. Vectors with a high clustering coefficient are the ones most likely to efficiently enact change to the adjacent vectors (Watts & Strogatz, 1998). Figure 7 shows the curriculum recalculated for clustering coefficient values.

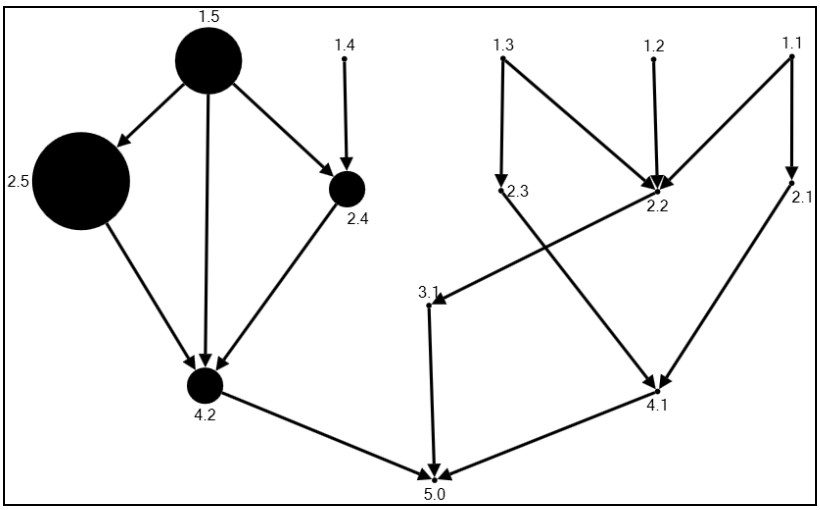

Figure 7: Clustering coefficient for the simple curriculum

When analyzing a curriculum, the clustering coefficient is useful in finding the course that can most efficiently implement change into adjacent courses. This is relevant because all assessment schemes require a feedback mechanism to modify the curriculum when the assessment data indicate a problem. The quicker this change can be incorporated into the curriculum, the sooner improved results will (hopefully) occur. Those courses with the highest clustering coefficient are the ones best suited for quick, efficient implementation.

In Figure 7, course 2.5 has the highest clustering coefficient in the curriculum. This implies that it is the best place to incorporate change quickly into the curriculum to correct anomalies indicated by assessment results. Course 1.5 has the second largest coefficient; therefore, it would also be a reasonable choice for change inclusion. However, given that the curriculum structure in Figure 7 is intended to mimic the four-year structure of a typical college curriculum, implementing change into 1.5 would take four years to reach the assessment point whereas using 2.5 would only take three years. Faster turnaround for the assessment feedback loop means that continuous improvement makes quicker progress; so course 2.5 is the preferred location.

Summary

This section has shown that an academic curriculum can be cast into the abstract representation of a graph. Once this is done, graph-theoretic metrics can be calculated on the structure of the curriculum graph to determine several interesting properties. Specifically, the out-degree can be used to locate those courses best suited to introduce topics and perform baseline assessment. The in-degree is useful in locating courses where exit assessment and higher-level learning activities should take place. The centrality measures of betweenness and eigenvector are valuable in identifying courses appropriate for assessment and topic reinforcement. Finally, the clustering coefficient can be used to find courses best suited to implement changes into a tightly connected clusters of courses. These metrics answer the four questions initially posed by this project. The next section will describe the results of applying this approach to an actual college curriculum.

APPLICATION OF THE APPROACH

The approach was applied to the curriculum of a college of business administration to determine its viability and effectiveness in a real situation. This college has six emphasis majors that include accounting, computer information systems (CIS), finance, general business, management, and marketing. One Bachelors degree is offered for all six emphasis areas and the entire program is accredited by the Association to Advance Collegiate Schools of Business (AACSB). The college is currently in the process of redesigning its assessment scheme, so it is an ideal test case for the technique. The remainder of this section will present an overview of the test; however, due to the extensive nature of the analysis and the limited space available, a detailed description of this application is not given in this paper, but will be the subject of future research.

The initial step was to build NodeXL models for each of the emphasis areas within the program. The graph for the CIS program is shown in Figure 8.

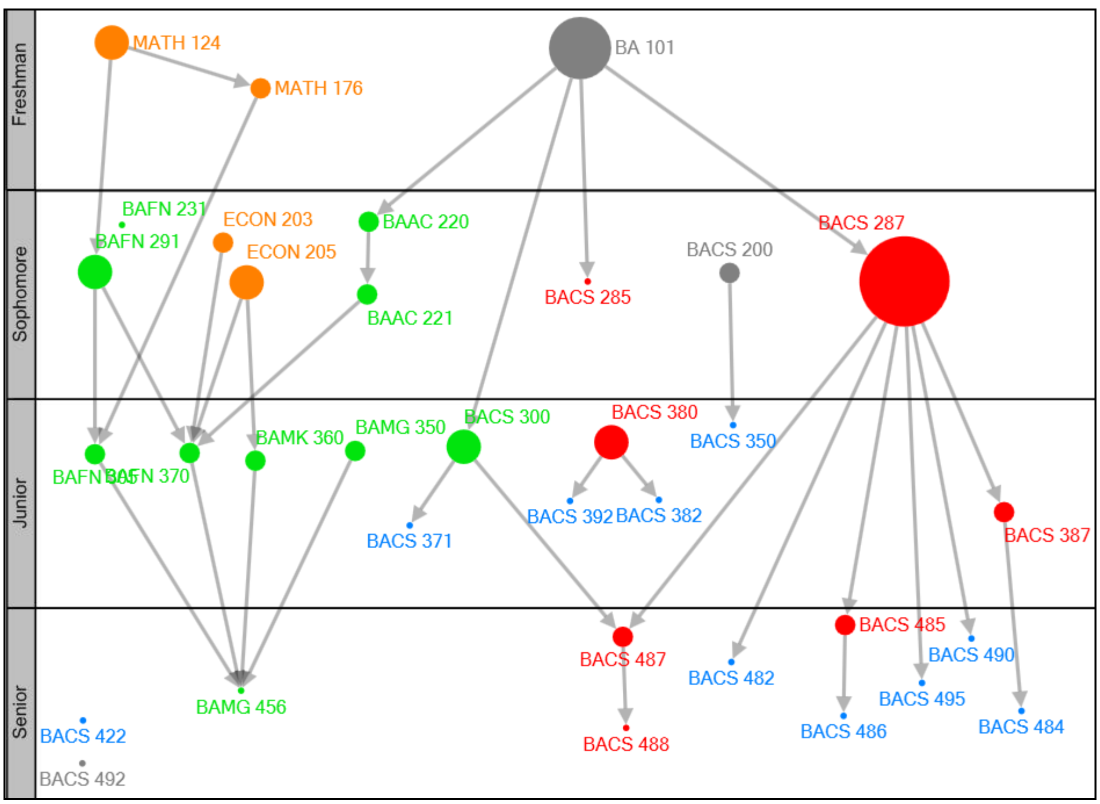

Figure 8: CIS program curriculum – out-degree emphasized

Two features of this of this graph are different from the previous examples. First, a "swim-lane" template was placed over the graph to indicate the program year when a student can first take a course. This helps align the curriculum with published advising literature. Second, different colors are used to indicate core required courses, elective emphasis courses, and non-business courses. In addition, Figure 8 shows the results of calculating the metrics and sizing the course vectors based on the out-degree of each course.

From this graph, it appears that BA 101 and BACS 287 are prime candidates to cover introductory topics in the curriculum. This accurately describes these courses as they are currently taught. In addition, they also could be used for baseline topic assessment.

While it is interesting to analyze individual emphasis areas with the technique, it is more useful to analyze the full curriculum. This was done by combining all six NodeXL spreadsheets and again running the graph metrics. Figure 9 shows the graph that this step produced.

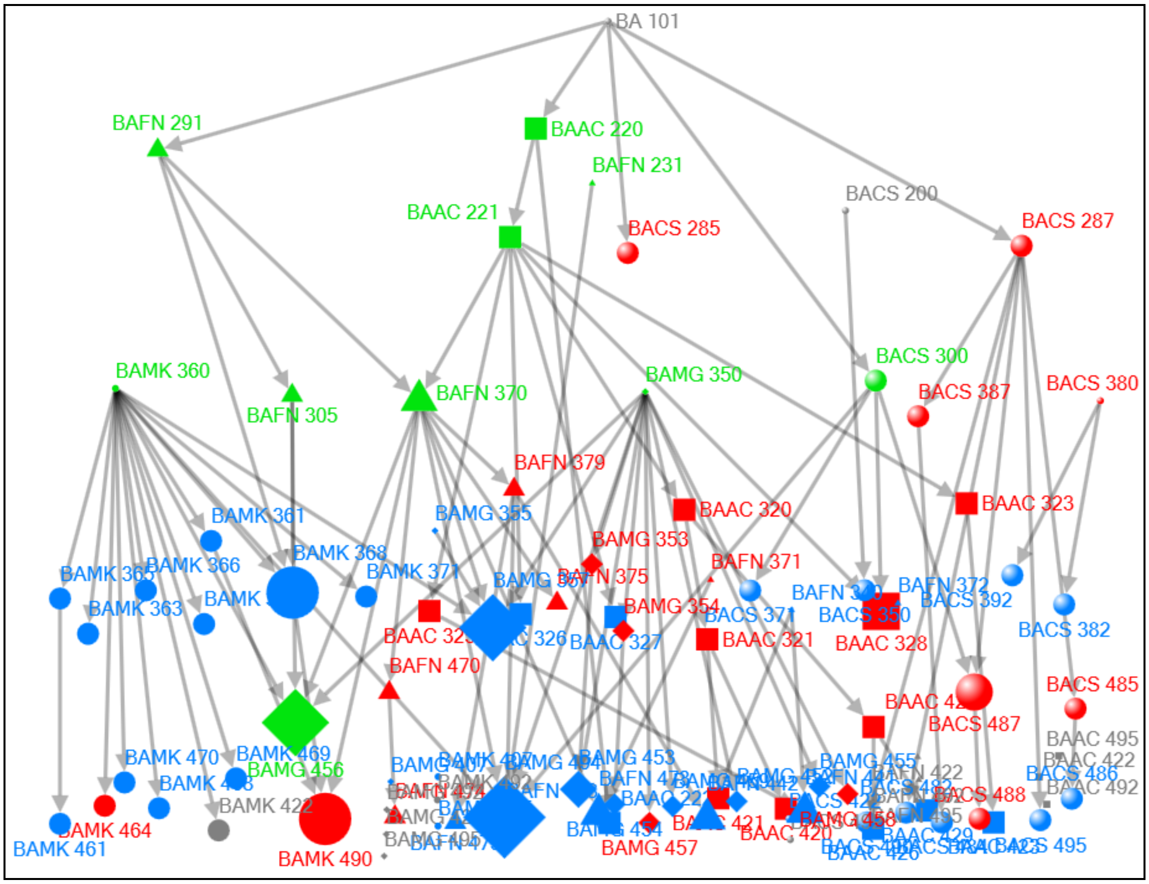

Figure 9: Full curriculum – in-degree emphasized

The swim-lane template was removed because it made the final graph too cluttered to be useful. The full curriculum graph also introduced the use of shaped vertices to represent the different emphasis areas; so round vertices represent marketing, square stand for accounting, and so on. This particular graph highlights the in-degree metric via the size of the course vertices.

Even with the removal of the swim-lane template, Figure 9 is obviously too complicated to interpret easily. To correct this, a NodeXL feature was utilized to selectively remove vertices and edges that meet user-specified criteria. Figure 10 shows the results of displaying only courses that are required by the curriculum.

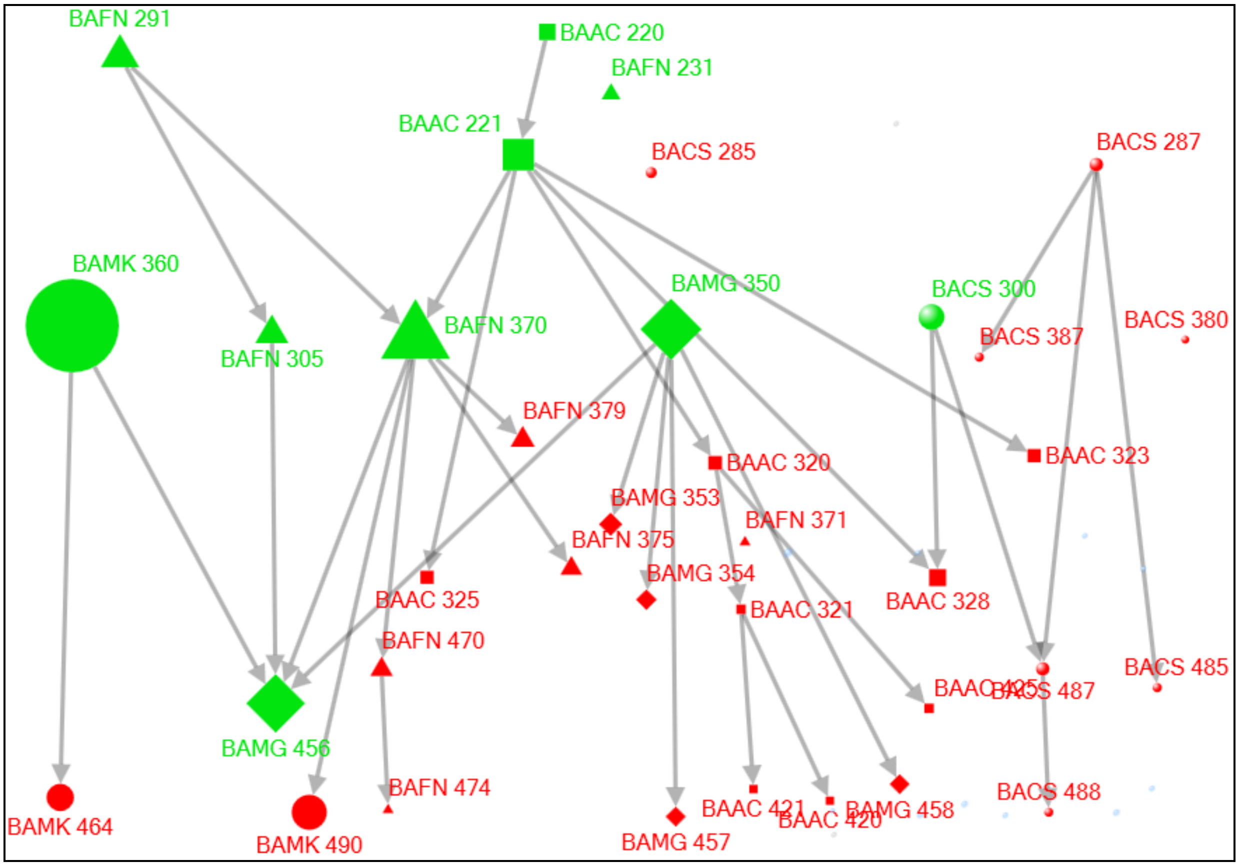

Figure 10: Full curriculum, required courses only – eigenvector centrality emphasized

This is a very practical simplification because most assessment schemes concentrate on the required courses in the curriculum since students taking electives tend to self-select. Figure 10 also emphasizes the eigenvector centrality metric, thereby indicating courses where topic reinforcement should take place prior to assessment. From this analysis, it appears that BAMK 360, BAFN 370, and BAMG 350 should all be involved in this activity. As a result of this analysis, these courses are being considered for this role in the college assessment scheme.

By systematically repeating the visualization of metric values and removing unnecessary graph clutter, an initial assessment scheme that takes the structure of the curriculum into account can be developed. This scheme analytically determines the placement of introductory topic coverage, the location for topic reinforcement, the courses where topics are assessed, and the courses where assessment changes are implemented. This initial assessment scheme can then serve as a starting point for the fine tuning and customization needed to build a truly effective assurance of learning scheme.

CONCLUSION

The current public mood demands that the money spent on education produce direct, measurable results in the form of improved student learning. This is evidenced by the recent standards required by accreditation bodies, reports published by the United States government, and the policies of school districts across the nation. Educational institutions have responded to this mandate by incorporating assessment of learning into their curricula. This activity is complex and typically produces many questions to which there are currently few answers. To mitigate this situation, this research addressed a facet of the problem concerning the placement of assessments, topic coverage, and corrective action within an existing curriculum.

The approach developed and described in this paper cast the curriculum problem into an abstract mathematical representation called a graph. Once this was done, graph-theoretic metrics were employed to analyze the curriculum structure and identify the appropriate courses to introduce topics, reinforce topics, and introduce topic changes. The metrics also indicated the appropriate placement of primary and secondary assessment within the curriculum. The technique uses Microsoft Excel and a freely available add-in, so little training is necessary and the cost is minimal. Finally, the approach has a flexible visualization component that generates a picture of the curriculum that is easy to understand and explain to non-mathematicians. The system was applied to the curriculum of an actual college and was shown to produce good results that are consistent and rational.

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